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Identification of elastic properties of laminates based on experiment design

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Abstract

A numerical–experimental method for identification of elastic properties of laminated composites from the experimental results is developed. It is proposed to use the method of experiment design and the response surface approach to solve the identification (inverse) problems. The response surface approximations are obtained by using the information on the behavior of a structure in the reference points of the experiment design. The finite element modeling of the structure is performed only in the reference points. Therefore, a significant reduction (about 50–100 times) in calculations of the identification functional can be achieved in comparison with the conventional methods of minimization. The functional to be minimized describes the difference between the measured and numerically calculated eigenfrequencies of structure. By minimizing the functional the identification parameters are obtained. It was shown that identification functional is convex if the stiffness matrix linearly depends on unknown parameters. The method is employed to identify the elastic properties of cross-ply laminates from the measured eigenfrequencies of composite plates. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Laminated composite; Identification; Experiment design

1. Introduction

Discussing the role of experimental mechanics (Knauss, 2000) in an increasing computational capabilities it was emphasized that now it is possible to extract physical information from more involved experiments that previously. In this direction during the last decade investigations for developing a new technique for material identification, the so-called mixed numerical–experimental technique, have started. Mainly stiffness properties have been investigated (Sol, 1986; Pedersen, 1989; De Wilde, 1991; Mota Soares et al., 1993; Grediac and Vautrin, 1993; Sol et al., 1993; Link and Zou, 1994; Frederiksen, 1997a).

The determination of stiffness parameters for complex materials such as fiber reinforced composites is much more complicated than for isotropic materials since composites are anisotropic and non-homogeneous. Conventional methods for determining stiffness parameters of the composite materials are based on direct measurements of strain fields, i.e. using information at a point of solid only. Boundary effects, sample

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size dependencies and difficulties in obtaining homogeneous stress and strain fields are some of the most serious problems. As a result of this, indirect methods using mainly a field information have recently received increasing attention. One such indirect method is based on measurements of the structure response and application of the numerical–experimental identification technique.

Mixed numerical–experimental methods are sensitive for model errors because the numerical model is always based on a series of hypotheses. If the real structure does not satisfy one or more of these hypotheses, the model of the structure is evidently not appropriate. Since the development of mixed numerical–experimental techniques for material identification is aimed at obtaining a practical method which yields quick and reliable results, much research has been done in order to minimize these model errors (Mota Soares et al., 1993; Frederiksen, 1997b,c).

In the meantime many different approaches were produced for identification of the physical parameters directly characterizing structural behavior (i.e. Young's modulus and density of the material). In Bolognini et al. (1993) appropriate comparison between actual eigenfrequencies of an existing structure and those obtained through the finite element analysis was performed. It led to the identification of parameters which can be used for the model calibration as well as for the detection of damaged zones in the structure.

Numerical–experimental identification methods are mainly used in structural applications. For example, in Mota Soares et al. (1993) elastic properties of laminated composites have been identified by using experimental eigenfrequencies. The stiffness parameters were identified from the measured natural frequencies of the laminated composite plate by direct minimization of the identification functional. Similar approach in order to identify the stiffness properties of the laminated composites was used in Frederiksen (1997a) and Araújo A.L. (1996). In Frederiksen (1997b) an improved plate model was used for identification of elastic constants of the laminate. In De Visscher et al. (1997) it was shown that the mixed numerical–experimental method can be used also for identification of damping properties of polymeric composites. Influence of modeling errors and measurement errors on parameter estimation was discussed in Frederiksen (1997b, 1998).

In the present study a numerical–experimental method for the identification of mechanical properties of laminated polymeric composites from the experimental results of the structure response has been further developed. The difference between conventional (Mota Soares et al., 1993) and present approach is that instead of direct minimization of identification functional the experiment design is used, by which response surfaces of the functional to be minimized are obtained. The response surface approximations are obtained employing the information on the behavior of a structure in the reference points of the experiment design. Note that the finite element modeling of the structure is performed only in the reference points. The functional to be minimized describes the difference between the measured and numerically calculated parameters of the response of structure. By minimizing the functional the identification parameters are obtained.

The main advantage of the present method is a significant reduction of the computational efforts. Previously this method based on experiment design and response surface approach was used for solution of the optimum design problems (Rikards, 1993; Rikards and Chate, 1995). Similarly, the response surface approach was used in design optimization (Roux et al., 1998). A review of optimization in relationship to experiment design was presented in Haftka et al. (1998).

2. Parameters of identification and criterion

The numerical–experimental method employed in the present study consists of the following stages. In the first stage the physical experiments are performed. Also the parameters to be identified, the domain of interest and criterion containing experimental data are selected. In the second stage the finite element method (FEM) is used in order to model the response of the structure and calculations are performed in a reference points of the domain of interest. The reference points are determined by using a method of experiment design. In the third stage the numerical data obtained by the finite element solution in the reference points are employed in order to determine approximating functions (response surfaces) for a calculation of the structure response. In the fourth stage, using the approximating functions and experimentally measured values of the structural response, the identification of the material properties is performed. For this a corresponding functional is minimized by using a conventional method of non-linear programing.

2.1. Parameters of identification

The present numerical–experimental approach is employed for identification of the elastic properties of laminated composite plates. For this the experimentally measured eigenfrequencies are used. It is assumed that the plate dimensions (see Fig. 1), plate mass, the layer angles β_i and the layer stacking sequence are known. Directions of material axes of a single layer are denoted 1–2–3, where 1 is the fiber direction and 2, 3 are the transverse directions. The unidirectional layer is assumed to be a homogeneous and transversely isotropic with respect to the fiber direction material.

The parameters to be identified are five elastic constants of the single layer of the laminate: E_1, E_2, G_{12}, G_{23}, ν_{12}. The laminated plate is modeled by the plate theory including transverse shear, rotatory inertia and extension of the normal line (Rikards and Chate, 1997). In this case the constitutive equation of the single layer of laminate in the symmetry axes of material is as follows

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 & 0 & 0 \\ A_{12} & A_{22} & A_{23} & 0 & 0 & 0 \\ A_{13} & A_{23} & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix}. \tag{1}$$

Here σ_{ij} is a stress tensor, ε_{ij} $(i, j = 1, 2, 3)$ is a strain tensor of the layer and A_{kl} $(k, l = 1, 2, \ldots, 6)$ is the elastic stiffnesses, which for the 3D stress state can be expressed through five elastic constants of the layer (Altenbach et al., 1996). In the case of plane stress state these relations are as follows

$$A_{11} = \frac{E_1}{1 - \nu_{12}E_2/E_1}, \quad A_{22} = \frac{E_2}{1 - \nu_{12}E_2/E_1}, \quad A_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}E_2/E_1}, \quad A_{44} = G_{23}, \quad A_{66} = G_{12}. \tag{2}$$

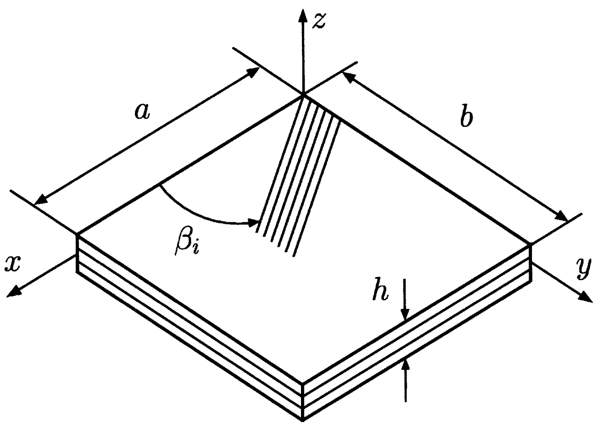

Fig. 1. Geometry of laminated plate.

Note that for transversely isotropic material $A_{22} = A_{33}$, $A_{12} = A_{13}$, $A_{55} = A_{66}$ and $A_{44} = \frac{1}{2}(A_{22} - A_{23})$.

Laminated plates investigated here is of moderate thickness. For such plates influence of extension of the normal line on eigenfrequencies is negligible. Sufficient accuracy can be achieved employing a Mindlin's plate theory accounting for the transverse shear deformations only. However, for the Mindlin's plate theory in the constitutive equation (1) instead of stiffnesses A_{ij} a reduced stiffnesses Q_{ij} are used (Reddy, 1996). In this case the reduced stiffnesses Q_{ij} are a non-linear functions of A_{ij}. The constitutive equation in form (1) is chosen to preserve linear dependence of the stiffness matrix of plate on parameters A_{ij}. Therefore, a convexity of identification problem can be proved (see below).

The plate is composed of unidirectionally reinforced layers. In general, the ith layer of the laminated plate can be oriented at an arbitrary angle β_i. The angles of the layers are assumed to be fixed. For example, the cross-ply laminate consists of the layers with angles $\beta_i = 0°$ and $\beta_i = 90°$.

The vector of parameters $\boldsymbol{x}$ to be identified can be chosen in a different ways. Components of this vector could be elastic engineering constants or elastic stiffnesses A_{ij}. The major problem in parameter estimation is ill-conditioning caused by unknown variables having substantially different order of magnitude. For example, Young's modulus and Poisson's ratio cannot be directly selected as components of vector $\boldsymbol{x}$ without proper scaling. In Frederiksen (1997c) the scaling by longitudinal modulus E_1 was employed and in addition a fixed scaling factor was chosen. Hence, all unknown parameters were measured on compatible scales. Similar scaling and reparametrisation was employed in Mota Soares et al. (1993), where additional scaling by the first experimental frequency allows to reduce the number of unknown variables from five to four. Thus, material parameters of the single layer can be expressed in terms of dimensionless variables α_i (Mota Soares et al., 1993)

$$
\begin{aligned}
\alpha_2 &= 4 - 4\left(\frac{E_2}{E_1}\right), \\
\alpha_3 &= 1 + \frac{E_2}{E_1}(1 - 2\nu_{12}) - 4\left(\frac{G_{12}}{E_1}\right)\alpha_0, \\
\alpha_4 &= 1 + \frac{E_2}{E_1}(1 + 6\nu_{12}) - 4\left(\frac{G_{12}}{E_1}\right)\alpha_0, \\
\alpha_5 &= 4(G_{23} + G_{12})\frac{\alpha_0}{E_1},
\end{aligned}
\tag{3}
$$

where

$$
\alpha_0 = 1 - \nu_{12}^2 \frac{E_2}{E_1}. \tag{4}
$$

The inverse relations of Eq. (3) are as follows

$$
\begin{aligned}
\frac{E_2}{E_1} &= \frac{4 - \alpha_2}{4}, \\
\frac{G_{12}}{E_1} &= \frac{8 - \alpha_2 - 3\alpha_3 - \alpha_4}{16\alpha_0}, \\
\nu_{12} &= \frac{\alpha_4 - \alpha_3}{8 - 2\alpha_2}, \\
\frac{G_{23}}{E_1} &= \frac{2\alpha_5 - 0.5(8 - \alpha_2 - 3\alpha_3 - \alpha_4)}{8\alpha_0}.
\end{aligned}
\tag{5}
$$

The variables α_i are chosen to be on compatible scales. The first set of identification variables is defined by these dimensionless quantities α_i. The vector $\boldsymbol{x}_{(1)}$ to be identified is given by

$$\boldsymbol{x}_{(1)} = [x_1, x_2, x_3, x_4] = [\alpha_2, \alpha_3, \alpha_4, \alpha_5] \tag{6}$$

In Mota Soares et al. (1993) additional scaling parameter C was introduced using the first experimental frequency in order to eliminate one variable. Let the experimental angular eigenfrequencies be designated by $\overline{\omega}_1, \overline{\omega}_2, \ldots, \overline{\omega}_I$, where I is the number of measured eigenfrequencies $\overline{f}_i$ ($\overline{\omega}_i = 2\pi\overline{f}_i$). The corresponding numerical (predicted) eigenfrequencies f_i ($\omega_i = 2\pi f_i$) for the set of material parameters α_i are represented by $\omega_1, \omega_2, \ldots, \omega_I$. Let us consider the scaling parameter C which is chosen according to the relation (Mota Soares et al., 1993)

$$C = \frac{\overline{\omega}_1^2}{\omega_1^2(E_1^0)}, \tag{7}$$

where ω_1 is the first numerical eigenfrequency calculated with the prior selected longitudinal Young's modulus E_1^0 of the layer (Mota Soares et al., 1993). In the present paper the initial guess value of the longitudinal modulus is taken $E_1^0 = 35$ GPa.

The second set of identification variables is defined by the stiffnesses A_{ij}. The vector $\boldsymbol{x}_{(2)}$ is given by

$$\boldsymbol{x}_{(2)} = [x_1, x_2, x_3, x_4, x_5] = [A_{11}, A_{22}, A_{12}, A_{44}, A_{66}]. \tag{8}$$

These parameters practically are to be on compatible scales. However, in order to obtain the engineering constants from A_{ij} in addition a system of three non-linear equations (2) should be solved.

The elastic constants can be evaluated through the identification procedure using the experimental eigenfrequencies of the laminated composite rectangular plate of constant thickness h, length a and width b (see Fig. 1). Identification procedure can be formulated using either the vector of unknown variables $\boldsymbol{x}_{(1)}$ or $\boldsymbol{x}_{(2)}$.

2.2. Identification functional and minimization problem

Firstly, the formulation of identification problem is given for the case if the vector of unknown variables is defined by Eq. (6).

The functional to be minimized describes deviation between the measured $\overline{\omega}_i$ and numerically calculated $\omega_i(\boldsymbol{x})$ frequencies (Mota Soares et al., 1993)

$$\Phi(\boldsymbol{x}_{(1)}) = \sum_{i=2}^{I} w_i^{(1)} \frac{(\overline{\omega}_i^2 - C[\omega_i(\boldsymbol{x}_{(1)})]^2)^2}{\overline{\omega}_i^4} = \sum_{i=2}^{I} w_i^{(1)} e_i^2. \tag{9}$$

Here e_i is relative discrepancy or residual and $w_i^{(1)}$ are weights for identification functional using the first set of variables. It is possible of assigning non-negative weights to each residual. For simplicity, only unity values are used hereafter. The estimation can be based on any set of frequencies by assigning weights of zeros and ones as appropriate.

The identification of the elastic constants $\boldsymbol{x}_{(1)}$ is performed on the basis of information obtained from the measurements of the I lowest frequencies. Note that the first frequency is not directly used for identification since this frequency was used for the scaling (see Eq. (7)). The identification problem is formulated as follows

$$\min_{\boldsymbol{x}_{(1)}} \Phi(\boldsymbol{x}_{(1)}) \tag{10}$$

subject to constrains

$$g_1(\boldsymbol{x}_{(1)}) = 4 - \alpha_2 > 0 \quad \text{or} \quad \frac{E_2}{E_1} > 0 \tag{11}$$

$$g_2(\boldsymbol{x}_{(1)}) = \frac{8 - \alpha_2 - 3\alpha_3 - \alpha_4}{16\left[1 - \left(\frac{\alpha_4 - \alpha_3}{8 - 2\alpha_2}\right)^2\left(\frac{4 - \alpha_2}{4}\right)\right]} > 0 \quad \text{or} \quad \frac{G_{12}}{E_1} > 0, \tag{12}$$

$$g_3(\boldsymbol{x}_{(1)}) = \frac{2\alpha_5 - 0.5(8 - \alpha_2 - 3\alpha_3 - \alpha_4)}{8\left[1 - \left(\frac{\alpha_4 - \alpha_3}{8 - 2\alpha_2}\right)^2\left(\frac{4 - \alpha_2}{4}\right)\right]} > 0 \quad \text{or} \quad \frac{G_{23}}{E_1} > 0, \tag{13}$$

$$g_4(\boldsymbol{x}_{(1)}) = -\left|\frac{\alpha_4 - \alpha_3}{8 - 2\alpha_2}\right| + \sqrt{\frac{4}{4 - \alpha_2}} > 0 \quad \text{or} \quad \sqrt{\frac{E_1}{E_2}} - |\nu_{12}| > 0, \tag{14}$$

$$\alpha_i^{\min} \leqslant \alpha_i \leqslant \alpha_i^{\max}, \quad i = 2, 3, 4, 5. \tag{15}$$

The upper $\alpha_i^{\max}$ and the lower $\alpha_i^{\min}$ bounds on the identification parameters can be chosen using preliminary information about the composite material. Constraints (11)–(14) denote conditions of a positive definiteness of the elasticity matrix.

In the second formulation of the identification problem the vector $\boldsymbol{x}_{(2)}$ (see Eq. (8)) is employed and the functional to be minimized is as follows

$$\Phi(\boldsymbol{x}_{(2)}) = \sum_{i=1}^{I} w_i^{(2)} \frac{\left(\overline{\omega}_i^2 - [\omega_i(\boldsymbol{x}_{(2)})]^2\right)^2}{\overline{\omega}_i^4} = \sum_{i=1}^{I} w_i^{(2)} e_i^2. \tag{16}$$

Here $w_i^{(2)}$ are weights for identification functional using the second set of variables. Again constrains for the upper and lower bounds on elastic constants are used and in addition constraints for positive definiteness of the elasticity matrix are employed

$$\begin{aligned} &g_1(\boldsymbol{x}_{(2)}) = A_{11} > 0, \qquad g_2(\boldsymbol{x}_{(2)}) = A_{22} > 0, \\ &g_3(\boldsymbol{x}_{(2)}) = A_{11}A_{22} - A_{12}^2 > 0, \\ &g_4(\boldsymbol{x}_{(2)}) = A_{44} > 0, \qquad g_5(\boldsymbol{x}_{(2)}) = A_{66} > 0. \end{aligned} \tag{17}$$

3. Vibration test of plates

The glass/epoxy cross-ply laminated plates consisting of eight unidirectional layers with a layer stacking sequence $[90/0/90/0]_s$ were tested. A geometric dimensions and density of plates are presented in Table 1. The eigenfrequencies of the test plates were measured by a real-time television (TV) holography. The samples were hung upon two threads in order to simulate free-free boundary conditions. The sample was located in front of the holographic testing device. A piezoelectric resonator (in the following called "sensor") was glued in one corner to excite the sample plate with increasing frequency. The sensor is of circular

Table 1
Parameters of the cross-ply laminated plates

Sample	a (m)	b (m)	h (mm)	ρ (kg/m^3)
PU10	0.1401	0.1401	2.011	1850
PU11	0.1401	0.1401	1.981	1850

shape with a diameter 25 mm located at the coordinates $x = a - 12.5$ mm and $y = b - 12.5$ mm. Mass of the sensor is $m_s = 3.5$ g.

The plate is illuminated by laser light and imaged by charged couple device (CCD) array, resulting in speckled image on the PC monitor. When the plate is deformed (exited), this interference pattern is slightly modified. Digital subtraction of two consecutive interference patterns yields a fringe pattern depicting the surface displacements of the plate. The nodal lines of the vibration modes can be easily identified on the monitor. The measurement technique more detailed was described in Yang et al. (1995) and Bledzki et al. (1999).

Experiments were performed for both plates considered (see Table 1) and about 20 flexural eigenfrequencies were measured. The mode shapes also were recognized in the experiment. In Table 2 the experimental plate flexural frequencies are presented. The quantity m denotes the wave number in the x direction and n denotes the wave number in the y direction (see Fig. 1). The mode shapes (nodal lines) for the plate PU10 are presented in Fig. 2. These mode shapes were recognized in the experiment and most of them were used in identification. However prior to experiment the mode shapes were calculated by the FEM using the initial guess values of the elastic constants. Such calculations is necessary in order to recognize all frequencies in the range and corresponding mode shapes. The mode shapes presented in Fig. 2 were calculated using the elastic constants obtained through identification (see Table 3 below). In calculations of the present mode shapes the mass of sensor was not taken into account. Calculating the plate with sensor the frequencies are slightly different and the mode shapes are similar but no more symmetric. Note that in the identification procedure each experimental frequency should be related to the numerical frequency having the same mode shape.

Table 2
Experimental plate flexural frequencies $\overline{f}_i$ (Hz)

Mode i	Mode shape m, n	Specimens	
		PU10	PU11
1	1, 1	166	159
2	2, 0	341	332
3	0, 2	–	–
4	2, 1	484	464
5	1, 2	542	529
6	2, 2	902	869
7	3, 0	971	938
8	3, 1	1090	1050
9	0, 3	1155	1143
10	1, 3	1273	1240
11	3, 2	1523	1470
12	2, 3	1643	–
13	4, 0	1898	1838
14	4, 1	2003	1940
15	3, 3	2290	2180
16	0, 4	–	–
17	4, 2	2418	2338
18	1, 4	–	–
19	2, 4	2733	2665
20	4, 3	–	–
21	5, 0	3108	3023
22	5, 1	3233	3135
23	3, 4	–	–
24	5, 2	3605	–

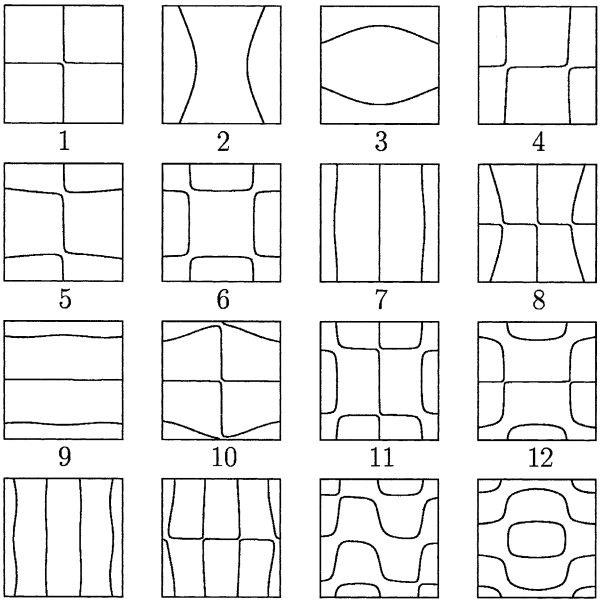

Fig. 2. Vibration modes of cross-ply laminated plate PU10.

Table 3
Elastic properties of single layer

Property	PU10		PU11	
	1	2	1	2
E_1, GPa	38.89	38.29	38.20	37.22
E_2, GPa	12.78	12.63	11.95	12.41
G_{12}, GPa	5.06	5.11	4.77	4.75
G_{23}, GPa	11.70	14.00	9.55	7.41
ν_{12}	0.304	0.350	0.392	0.426

4. Finite element solution

The eigenvalue problem for harmonic vibrations of the plate can be represented by

$$\boldsymbol{K}\boldsymbol{u} = \omega^2 \boldsymbol{M}\boldsymbol{u}. \tag{18}$$

Here $\boldsymbol{K}$ is the stiffness matrix of the plate, $\boldsymbol{M}$ is the mass matrix and $\boldsymbol{u}$ is the displacement vector. The eigenvalue relation (18) equation for the mode $\boldsymbol{u}_1$ which corresponds to the first experimental eigenfrequency $\overline{\omega}_1$ can be written in an equivalent form placing E_1 in evidence

$$E_1 \boldsymbol{K}^* \boldsymbol{u}_1 = \overline{\omega}_1^2 \boldsymbol{M}\boldsymbol{u}_1. \tag{19}$$

Here $E_1 \boldsymbol{K}^* = \boldsymbol{K}$ is the stiffness matrix. Taking into account relation (7) this equation can be written as

$$C E_1^0 \boldsymbol{K}^* \boldsymbol{u}_1 = C \omega_1^2 \boldsymbol{M}\boldsymbol{u}_1, \tag{20}$$

hence

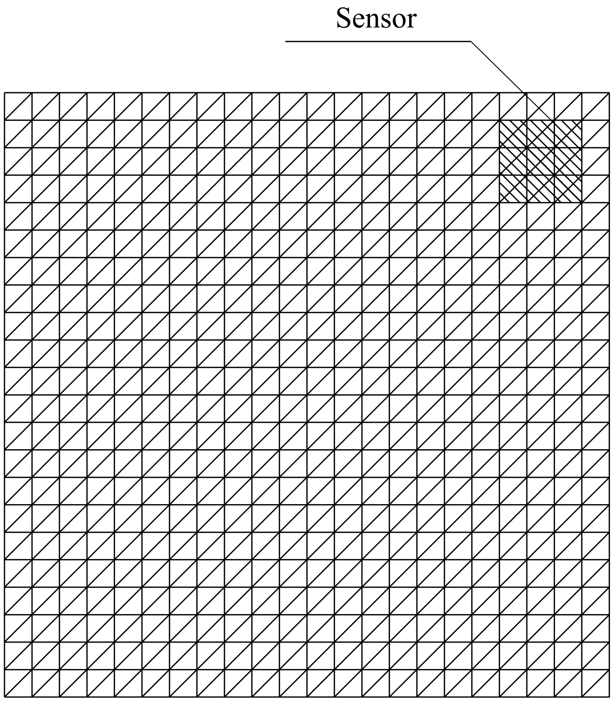

Fig. 3. Finite element mesh.

$$E_1 = CE_1^0, \tag{21}$$

where E_1^0 is the initial guess value given to the Young's modulus in the fiber direction of the layer and E_1 is the corresponding identified mechanical property. After evaluation of the optimum value of $\boldsymbol{x}_{(1)}$ (the first set of identification variables) the remaining mechanical properties are calculated by inverse relations (5).

For the second set of identification variables $\boldsymbol{x}_{(2)}$ the same eigenvalue equation (18) has been solved. The only difference is that other reference points of experiment design (see below) are selected since in this case the dimension of the design space is five. Hence numerical frequencies are predicted in other points of the design space as in the case of using the vector $\boldsymbol{x}_{(1)}$, for which the dimension of the design space is four.

The eigenvalue problem (18) was solved by using subspace iteration technique (Bathe, 1982) and a triangular finite element of laminated thick plate with a shear correction (Rikards and Chate, 1997). In order to avoid 'shear locking' a selective integration technique was applied. A 22×22 regular mesh (968 finite elements) was considered in order to achieve appropriate accuracy for at least 20 first eigenvalues of the laminated plate with FFFF (all edges free) boundary conditions. The finite element mesh of the plate is shown in Fig. 3. In calculations of predicted frequencies $\omega(\boldsymbol{x}_{(1)})$ or $\omega(\boldsymbol{x}_{(2)})$ the mass of sensor should be taken into account.

5. Method of experiment design

In this paper the approach suggested (Audze and Eglais, 1977) and used later in solution of optimal design problems (Rikards, 1993) is followed. It considers a non-conventional criterion for elaboration of plans of experiments which is not dependent on the mathematical model (approximating functions) under consideration. The input data for the elaboration of the plan include only the number of variables n (number of factors) and the number of experiments k. The main principles in the present approach are as follows

1. The number of levels of factors (same for each factor) is equal to the number of experiments and for each level there is only one experiment.
2. The reference points (points of experiment) are distributed as regular as possible in the domain of variables.

To realize the second principle it is suggested to use a criterion

$$\Psi = \sum_{i=1}^{k-1}\sum_{j=i+1}^{k}\frac{1}{L_{ij}^2} \Rightarrow \min, \tag{22}$$

where L_{ij} is a distance between the reference points having numbers i and j ($i \neq j$). The criterion (22) is a physical analogy of the minimum of potential energy of repulsive forces for a set of points of unit mass, if the magnitude of these repulsive forces is inversely proportional to the distance L_{ij} cubed between the points. Note that the force can be obtained as derivative of the potential energy.

The problem of minimizing the criterion (22) together with the first principle leads to a non-linear programming problem. Solving the non-linear programming problem the plans of experiments were determined for different number of the design variables n and different number of the experiments k.

The plan of experiment is characterized by a matrix B_{ij}, which contains the levels of factors for each of k experiments. For example, for a number of design variables (factors) $n = 2$ and $k = 9$, the matrix of plan is given by

$$\boldsymbol{B}^{\mathrm{T}} = \begin{bmatrix} 7 & 1 & 2 & 5 & 4 & 9 & 6 & 8 & 3 \\ 2 & 6 & 3 & 5 & 1 & 4 & 9 & 7 & 8 \end{bmatrix}. \tag{23}$$

The corresponding plan of experiments is shown in Fig. 4. The domain of interest (domain of variables) is determined as $x_j \in [x_j^{\min}, x_j^{\max}]$, where $x_j^{\min}$ and $x_j^{\max}$ are respectively the lower and the upper bounds on the

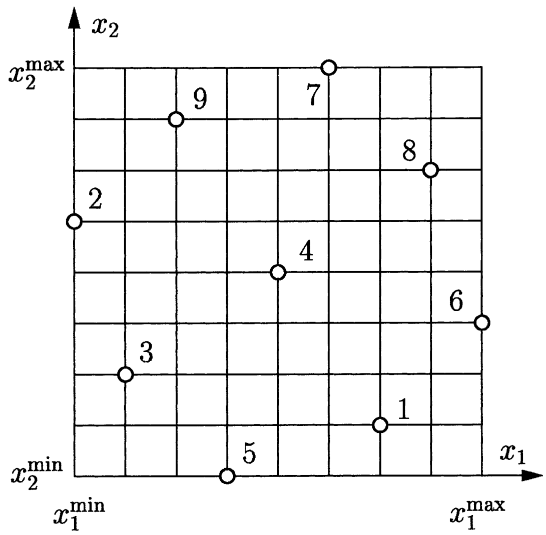

Fig. 4. Experiment design for $n = 2$ and $k = 9$.

design variables. Thus, in this domain the reference points, where the numerical experiments must be performed, are calculated by the expression

$$x_j^{(i)} = x_j^{\min} + \frac{1}{k-1}(x_j^{\max} - x_j^{\min})(B_{ij} - 1). \tag{24}$$

Here $i = 1, 2, \ldots, k$ and $j = 1, 2, \ldots, n$. Since the matrices B_{ij} of the experiment design are universal, they may be used for the various identification or optimum design problems. Some other approaches in relationship with applications of the experiment design in solution of optimization problems were reviewed in Haftka et al. (1998).

6. Approximation of the response surface

Methods based on approximation concepts take dominant position in design optimization and the development of new high quality approximation functions has become a separate problem (Van Campen et al., 1990; Barthelemy and Haftka, 1993). Approximation models are often used in engineering optimization when the optimization using the original function (original simulation model) is expensive.

The most popular approximating models are polynomial models, created by performing a least squares curve fit to a set of data. The data consist of response values at the selected number of reference points. These points can be selected using different experiment designs. The polynomial based modeling methods have come to be known as response surface (RS) models.

Techniques from experiment design and response surface methodology (Box et al., 1987) were used to build the approximate models from the data in the reference points. In (Roux et al., 1998) and other investigations the response surface methodology was applied for solution of optimum design problems. There are different ways of selection of approximating functions other than polynomials. Recently the response surfaces for optimization problems were obtained employing genetic programming (Toropov et al., 1998). Other possibility to build a model is using engineering knowledge of the true functional form of the response (Vanderplaats, 1984).

The RS methodology can be used also for solution of identification problems. Consider a response y (in our case eigenfrequencies) dependent on a set of variables $\boldsymbol{x}$. In our case there are four (the first set of identification variables) or five (the second set of identification variables) identification variables representing the elastic constants of the material. Thus the original function is denoted by

$$y = y(\boldsymbol{x}). \tag{25}$$

The original function in our case is determined through deterministic computer simulations (finite element solutions). Since the original function is computationally expensive the approximation is employed. The approximating function is defined by

$$\tilde{y} = \tilde{y}(\boldsymbol{x}). \tag{26}$$

Note that approximating function is valid only at the some domain of interest defined by upper and lower bounds on the variables and in our case in addition defined by positive definiteness of the elasticity matrix. Important is choice of upper and lower bounds on the variables since using smaller subregions can increase accuracy of approximation. Further it will be shown (see below) that the original function (eigenvalue) is concave function in the design space defined by the second set of identification variables.

The selection of points in the design space where the response must be evaluated was discussed in the previous section. The goal is by using the data only in the reference points (in our case these data are obtained by the finite element solution of the eigenvalue problem (18)) to obtain the approximation $\tilde{y}(\boldsymbol{x})$ in the mathematical form. Here such mathematical models (response surfaces) have been obtained for the first I eigenfrequencies of the cross-ply laminated plates.

The existing methods of regression analysis are based on the principle that the equation form is a priori known and the problem is to find coefficients (the so-called tuning parameters) of the equation. However, in most cases the form of the equation also must be determined. Such a method was proposed earlier (Rikards, 1993; Rikards and Chate, 1995) to obtain a simple mathematical models for the structural optimization problems. Further this method is briefly outlined and applied for the identification problem considered here.

Let us consider a method, in which a form of the regression equation is unknown. There are two requirements for the regression equation – accuracy and reliability. Accuracy is characterized by a minimum of standard deviation. Increasing the number of terms in the regression equation (response surface) could improve accuracy, moreover, it is possible to obtain a complete agreement between the experimental data and values given by the equation of regression. But in this case the prediction can be very poor at the other points of the domain of interest. In Toropov et al. (1998) where the approximating function was built using genetic programming the quality of approximation is measured by the fitness function and special coefficient penalizing the excessive length of the expression was introduced. Reliability of the regression equation means that accuracy for the reference points and any other point in the domain of interest is approximately the same. Obviously, if the number of terms of the regression equation decreases, the reliability of the model increases. A compromise between accuracy and reliability of the model must be found.

The selection of the 'best' regression equation (response surface) in the subregion defined by the lower and upper bounds on the design variables is performed by the following procedure. The response surface is built by using data obtained by computer simulation in all points of experiment design. First, consider the approximating function (model) of the following form

$$\tilde{y}(\boldsymbol{x}) = \sum_{i=1}^{m} A_i \phi_i(\boldsymbol{x}), \tag{27}$$

where A_i are unknown coefficients (tuning parameters) and $\phi_i(\boldsymbol{x})$ are the functions that constitute the model. These functions are built from the set of simple functions $\varphi_1, \varphi_2, \dots, \varphi_R$. The functions φ_r $(r = 1, 2, \dots, R)$ are assumed to be in the form

$$\varphi_r(\boldsymbol{x}) = \prod_{j=1}^{n} x_j^{\alpha_{rj}}, \tag{28}$$

where n is the number of variables and α_{rj} are positive or negative including zero integers. The form of the approximating function (27) is determined in two steps. First, the perspective functions $\phi_i(\boldsymbol{x})$ are selected by using the least-squares estimation. Then a step-by-step reduction procedure of the number of terms in the model is applied and further reduction of the selected functions is performed. In each step a new set of tuning parameters A_i is obtained. Details of this procedure and a corresponding program called RESINT were described in Rikards (1993). The reduction procedure for one numerical example will be shown below in order to obtain the approximating function for the second eigenvalue of Eq. (18) describing the vibrations of the cross-ply laminated plate. Note that there is no general rules for the procedure of reduction of terms in the model (response surface function) and it is necessary to acquire some experience to obtain appropriate function.

Approximating functions for the functionals (9) and (16) are obtained as follows. For the first set of variables (6) the approximations are obtained for the frequencies $f_i = \omega_i/2\pi$. The approximating functions $\tilde{f}_i(\boldsymbol{x}_{(1)})$, i.e. $\tilde{\omega}_i(\boldsymbol{x}_{(1)}) = 2\pi\tilde{f}_i(\boldsymbol{x}_{(1)})$, then are used in minimization process of the functional (9) instead of original functions. For the second set of variables (8) the approximations are obtained for the eigenvalues

$\lambda_i = \omega_i^2$. The approximating functions $\tilde{\lambda}_i(\boldsymbol{x}_{(2)})$ then are used in minimization process of the functional (16) instead of original functions.

7. Determination of elastic constants

Let us consider the procedure of identification for the cross-ply composite plates. The experiment design with four variables ($n = 4$) and 35 reference points ($k = 35$) was selected for the first set of variables Eq. (6). The experiment design with five variables ($n = 5$) and 36 reference points ($k = 36$) was selected for the second set of variables Eq. (8).The upper and lower bounds (domain of interest) of the elastic constants for the first set of variables were taken as follows

$$\begin{aligned} &5\ \text{GPa} \leqslant E_2 \leqslant 20\ \text{GPa},\\ &3\ \text{GPa} \leqslant G_{12} \leqslant 9\ \text{GPa},\\ &3\ \text{GPa} \leqslant G_{23} \leqslant 14\ \text{GPa},\\ &0.2 \leqslant v_{12} \leqslant 0.4. \end{aligned} \tag{29}$$

The upper and lower bounds of the elastic modulus E_1 for the second set of variables were selected by

$$33\ \text{GPa} \leqslant E_1 \leqslant 43\ \text{GPa}. \tag{30}$$

Constraints for other elastic constants were taken the same Eq. (29) as for the first set of variables.

For the first set of variables Eq. (6) the lower and upper bounds were recalculated by Eq. (3). For the second set of variables Eq. (8) the lower and upper bounds were recalculated by Eq. (2). By using the matrix B_{ij} of the experiment design ($n = 4$ and $k = 35$ for the first set of variables, $n = 5$ and $k = 36$ for the second set of variables) in the expression (24) the values of $\boldsymbol{x}_{(1)}$ and $\boldsymbol{x}_{(2)}$ were calculated for all reference points. Employing the values of parameter $\boldsymbol{x}_{(1)}$ in the reference points and the initial guess value $E_1^0 = 35$ GPa the first 20 natural frequencies in all k points were calculated. Similar calculations were performed also for the second set of variables $\boldsymbol{x}_{(2)}$.

The finite element mesh of the plate was shown in Fig. 3. It should be noted that there is some originality in calculation of the mass matrix $\boldsymbol{M}$ in Eq. (18). In order to represent more accurate an inertia forces of the plate, the mass of sensor m_s should be taken into account. In the finite element modeling it is assumed that the finite elements where the sensor is located (see Fig. 3) have the same thickness h as the plate, but for these finite elements an equivalent density ρ_{eqv} is calculated

$$\rho_{eqv} = \rho + \frac{m_s}{F_s h}.$$

Here F_s is the area of the sensor.

The data of numerical simulations were used to determine the response surfaces (27). For this the RESINT program (Rikards, 1993) was employed. For the first set of variables approximating functions were obtained for frequencies. These functions were used in the functional (9). For the second set of variables approximating functions were obtained for the eigenvalues. These functions were used in the functional (16).

For the second set of variables $\boldsymbol{x}_{(2)}$ as example two approximating functions (first and second eigenvalue) are given below

$$\begin{aligned} \lambda_1(\boldsymbol{x}_{(2)}) &= 0.127 \times 10^7 + 0.602 \times 10^6 z_5 + 0.155 \times 10^5 z_2,\\ \lambda_2(\boldsymbol{x}_{(2)}) &= 0.456 \times 10^7 + 0.126 \times 10^7 z_2 + 0.362 \times 10^6 z_1 - 0.304 \times 10^6 z_3\\ &\quad + 0.240 \times 10^5 z_5 - 0.133 \times 10^6 z_3^2 + 0.443 \times 10^5 z_1 z_3. \end{aligned} \tag{31}$$

Here normalized variables are introduced

$$\begin{aligned} z_1 &= -8.21 + 0.21 \times 10^{-9} A_{11}, \\ z_2 &= -1.61 + 0.12 \times 10^{-9} A_{22}, \\ z_3 &= -1.46 + 0.31 \times 10^{-9} A_{12}, \\ z_4 &= -1.55 + 0.18 \times 10^{-9} A_{44}, \\ z_5 &= -2.00 + 0.33 \times 10^{-9} A_{66}. \end{aligned} \quad (32)$$

Correlation c of approximating functions with the FEM data for the first eigenvalue is $c = 0.979$ and for the second eigenvalue $c = 0.991$. Similar expressions were obtained for other eigenvalues. Note that transverse shear modulus is presented only in approximations for the higher eigenvalues, i.e. for the modes 9, 10 and 13. Influence of the transverse shear deformations on the first frequencies is small and therefore this variable is not presented in the all approximating functions.

The approximating functions were obtained using the step-by-step elimination procedure. The diagram of reduction of terms building the function for the second eigenvalue is shown in Fig. 5. It is seen that the first break in diagram corresponds to the regression equation with seven terms. At eliminating the seventh term the correlation with the data of numerical experiment decreases much more in comparison with the previous step. Eliminating further this term an accuracy of approximation can be lost. Thus, for the second eigenvalue the approximating function is selected with seven terms. It should be noted that polynomial terms were not selected a priori. These terms were obtained in the process of building the model and selected as the best regressors.

Depending on approximations in the identification different number of experimental eigenfrequencies were used. Thus, for the plate PU10 using the first set of variables in identification procedure the vector of weights (see Eq. (9)) is as follows

$$\mathbf{w}^{(1)} = [0 \ 1 \ 0 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0]$$

Note, that using in identification the first set of variables the first experimental frequency is taken into account indirectly, i.e. in the scaling (see Eq. (7)) but not directly in the functional Eq. (9). Thus, in this case at all 14 experimental frequencies are employed in identification.

For the same plate PU10 using the second set of variables in identification procedure the vector of weights (see Eq. (16)) was set different

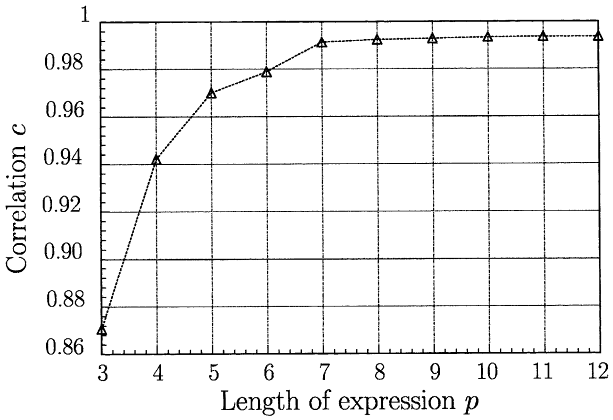

Fig. 5. Diagram of term reduction for the function $\lambda_2(\mathbf{x}_{(2)})$.

$$\boldsymbol{w}^{(2)} = [1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0].$$

Thus, using for identification the second set of variables only seven experimental frequencies are employed. Such difference is due to different approximating functions. In the present paper only the approximating functions with correlation $c > 0.90$ are employed.

For the plate PU11 using the first set of variables in identification procedure the vector of weights is as follows

$$\boldsymbol{w}^{(1)} = [0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0].$$

In this case at all 12 experimental frequencies are employed in identification. For the same plate PU11 using the second set of variables in identification procedure the vector of weights is given by

$$\boldsymbol{w}^{(2)} = [1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0].$$

In this case for identification only eight experimental frequencies are employed.

Minimization of the functional (9) or (16) subject to constrains was performed by the random search method (Rikards, 1993). For the second set of variables in addition a system of three non-linear equations (2) was solved. Results of identification of the layer stiffness properties for the two plates considered (PU10 and PU11) are presented in Table 3. Here in the columns noted by 1 and 2 the results obtained using in identification the first and second set of variables are presented.

It is seen that in-plane Young's moduli E_1 and E_2 and shear modulus G_{12} obtained by using different set of variables are in good agreement. Larger differences is for the Poisson's ratio ν_{12}. The reason is that influence of Poisson's ratio on frequencies is considerably smaller and in this case accuracy of approximation is not sufficient. The transverse shear modulus G_{23} is overestimated. The transverse shear modulus cannot be reliable determined from the measured frequencies since the plates were too thin $(h/a = 1/70)$ for identification of this property. In this case more thick plates should be used.

Verification of the results was performed by the FEM and through the independent experiments. For the finite element analysis the elastic constants obtained by the identification procedure were used (see Table 3). Results are shown in Table 4. Residuals were calculated by the expression

$$\Delta_i = \frac{f_i^{\mathrm{FEM}}(\boldsymbol{x}^*) - f_i^{\exp}}{f_i^{\exp}} \times 100. \tag{33}$$

It is seen that differences between the experimental and numerical frequencies calculated by using elastic constants obtained by identification are very small. Mostly residuals do not exceed 1%. Exception is for the mode 15 of the plate PU10 since in this case the difference is 3.14%. Note that for the verification at all 19 (plate PU10) and 17 (PU11) experimental frequencies are used but for the identification considerably smaller number of frequencies were taken into account. Predicted frequencies (see Table 4) which were not taken into account for identification also are in good agreement with the experimental ones.

It is of interest to compare the elastic constants of the single layer of the cross-ply laminate with the properties obtained for the unidirectionally (UD) reinforced transversely isotropic plate made from the same material (Bledzki et al., 1999). Results are presented in Table 5. For the cross-ply laminate the mean values calculated from four values of Table 3 are showed. Good agreement of the results for the constants E_1, E_2 and G_{12} is observed. However, there is some difference for the Poisson's ratio. It can be explained since this property is less sensitive to frequencies as modulus of elasticity, especially for the cross-ply laminate. Due to approximations an accuracy for the Poisson's ratio has been lost. In Mota Soares et al. (1993) it was shown that good accuracy for the Poisson's ratio can be obtained employing in minimization the original (numerical) functions instead of approximating functions.

Table 4
Flexural frequencies f_i (Hz) and residuals Δ_i for plates PU10 and PU11

Mode i	PU10			PU11		
	Experimental	FEM	Δ_i (%)	Experimental	FEM	Δ_i (%)
1	166	166.4	+0.24	159	158.5	−0.31
2	341	344.2	+0.94	332	332.1	+0.03
3	–	416.2	–	–	407.3	–
4	484	486.1	+0.43	464	470.1	+1.31
5	542	545.9	+0.72	529	526.9	−0.40
6	902	895.6	−0.71	869	866.8	−0.25
7	971	967.5	−0.36	938	944.0	+0.64
8	1090	1087	−0.28	1050	1050	0
9	1155	1165	+0.87	1143	1134	−0.79
10	1273	1278	+0.39	1240	1247	+0.56
11	1523	1513	−0.66	1470	1469	−0.07
12	1643	1632	−0.67	–	1584	–
13	1898	1865	−1.74	1838	1814	−1.30
14	2003	2001	−0.10	1940	1950	+0.51
15	2290	2218	−3.14	2180	2155	−1.15
16	–	2275	–	2212	–	
17	2418	2379	−1.61	2338	2314	−1.03
18	–	2414	–	–	2355	–
19	2733	2724	−0.33	2665	2657	−0.30
20	–	3010	–	–	2923	–
21	3108	3157	+1.58	3023	3074	+1.69
22	3223	3226	+0.22	3135	3141	+0.19
23	–	3315	–	–	3230	–
24	3605	3617	+0.33	–	3522	–

Table 5
Comparison of results for cross-ply and UD laminates

Property	Cross-ply plate	UD plate
E_1, GPa	38.15	38.81
E_2, GPa	12.44	12.12
G_{12}, GPa	4.92	5.09
ν_{12}	0.368	0.255

8. Convexity of identification problem

It can be proved the uniqueness of solution of the problem of identification of elastic constants. Actually, the objective function $\Phi(\boldsymbol{x})$ in Eq. (16) is convex if the eigenvalues $\lambda(\boldsymbol{x})$ is a concave functions of $\boldsymbol{x}$ since in the objective function the predicted eigenvalue $\lambda(\boldsymbol{x})$ is with the minus sign. The proof of this statement is as follows.

The eigenvalue problem (18) can be written in the form

$$\boldsymbol{K}(\boldsymbol{x})\boldsymbol{u} = \lambda(\boldsymbol{x})\boldsymbol{M}\boldsymbol{u}, \quad \lambda(\boldsymbol{x}) = \omega^2(\boldsymbol{x}). \tag{34}$$

Here the $l \times l$ stiffness matrix $\boldsymbol{K}$ is a linear function of the parameters of identification $\boldsymbol{x}$ if the constitutive matrix of material Eq. (1) linearly depends on unknown variables. Matrices $\boldsymbol{K}$ and $\boldsymbol{M}$ are a symmetric positive definite matrices. The matrix $\boldsymbol{K}$ depends on parameter $\boldsymbol{x}$, while the matrix $\boldsymbol{M}$ is independent of it.

Vector $\boldsymbol{u} \in E^l$ characterizes all possible vibration modes. In Gantmacher (1959) it was shown that the extreme eigenvalue of the problem (34) can be determined through the relation

$$\lambda_{\min} = \min(\boldsymbol{Ku}, \boldsymbol{u}), \ \boldsymbol{u} \in E^l, \ (\boldsymbol{Mu}, \boldsymbol{u}) = 1. \tag{35}$$

Here $(\boldsymbol{Ku}, \boldsymbol{u})$ is notation for the quadratic form. Now the following property of concave functions is employed (Rockafellar, 1970). Assume that $d_i(\boldsymbol{x})$ $(i = 1, 2, \ldots, m)$ are concave functions of parameter $\boldsymbol{x}$. Then the function

$$d(\boldsymbol{x}) = \min[d_1(\boldsymbol{x}), d_2(\boldsymbol{x}), \ldots, d_m(\boldsymbol{x})] \tag{36}$$

is also concave, i.e., the minimum of set of concave functions of parameter $\boldsymbol{x}$ forms a concave function of $\boldsymbol{x}$.

For all possible vectors $\boldsymbol{u}$ the quadratic form $(\boldsymbol{Ku}, \boldsymbol{u})$ in Eq. (35) forms a set of functions of parameter $\boldsymbol{x}$. Here vectors $\boldsymbol{u}$ in Eq. (35) are arbitrary in E^l under condition that they are orthonormalized in $\boldsymbol{M}$-metric, i.e., $(\boldsymbol{Mu}, \boldsymbol{u}) = 1$.

The conditions described above can be summarized in the following statement. The lowest eigenvalue of the problem $\boldsymbol{Ku} = \lambda\boldsymbol{Mu}$, where $\boldsymbol{K}$ is a symmetrical positive definite matrix that depends on parameter $\boldsymbol{x}$, and $\boldsymbol{M}$ is a symmetric positive definite matrix that is independent of $\boldsymbol{x}$, is a concave function of $\boldsymbol{x}$ if quadratic form $(\boldsymbol{Ku}, \boldsymbol{u})$ is a concave function of $\boldsymbol{x}$. This statement firstly was proved in connection with solution of the optimum design problems of composite shells (Rikards, 1980).

From the above statement it follows that if the elements of $\boldsymbol{K}$ are linear functions of $\boldsymbol{x}$, then the quadratic form $(\boldsymbol{Ku}, \boldsymbol{u})$ is also a linear function of $\boldsymbol{x}$. Since a linear function is simultaneously concave and convex, the lowest eigenvalue $\lambda_{\min}$ of the problem $\boldsymbol{Ku} = \lambda\boldsymbol{Mu}$ is a concave function of the parameter $\boldsymbol{x}$. Similarly can be proved a statement that the second, third and higher eigenvalues are also concave functions of parameter $\boldsymbol{x}$. For this a condition of orthogonality of eigenvectors should be used (Gantmacher, 1959).

The properties of eigenvalues can also be analyzed numerically. From analysis of contour plots of the eigenvalues it is seen that these are concave functions of parameter $\boldsymbol{x}$. Identification functional (16) is a quadratic function of predicted eigenvalues $\lambda(\boldsymbol{x})$. Since $\lambda(\boldsymbol{x})$ in the functional is with the minus sign the function in brackets is convex. Thus the functional (16) is a convex function of parameter $\boldsymbol{x}$. It is seen in Fig. 6, where the contour plot of the identification functional for the plate PU10 is presented.

Due to convexity of the functional more simple approximating functions can be obtained building a model for the response surface method. Thus, minimization employing the simplified model is computationally much less expensive than using the original (numerical) functions.

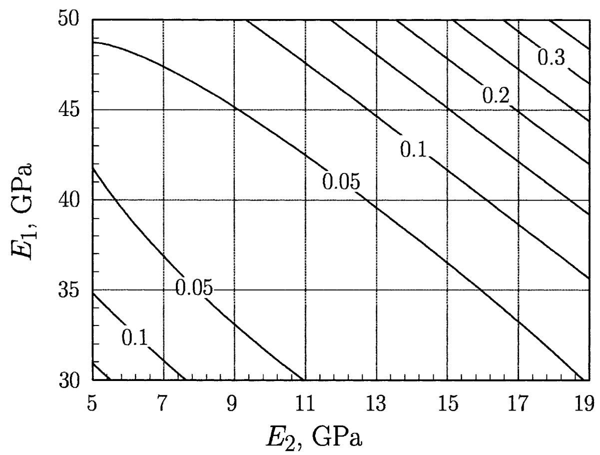

Fig. 6. Contour plot of identification functional Eq. (16) for plate PU10.

9. Conclusions

Elastic constants of a single layer of the cross-ply laminate have been determined by using the identification procedure based on the method of experiment design and the response surface approach. It was shown that identification functional is convex if the stiffness matrix linearly depends on unknown parameters.

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